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1995 J. Phys. A: Math. Gen. 28 L453

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LETTER TO THE EDITOR

On the dynamics of a spherical spin-glass in a magnetic field

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Received 5 June 1995

Abstract. We carry out an analysis of the effect of a quenched magnetic field on the dynamics of the spherical Sherrington–Kirkpatrick spin-glass model. We show that there is a characteristic time introduced by the presence of the field. First, for times sufficiently small the dynamic scenario of the zero field case—with aging effects—is reproduced. Second, for times larger than the characteristic time one sees equilibrium dynamics. This dynamical behaviour is reconciled with the geometry of the energy landscape of the model. We compare this behaviour with experimental observations at a finite applied field.

For some time there has been much interest in aging phenomena and in general non-equilibrium behaviour [1]. For a wide range of models one finds that if the thermodynamic limit is taken before the long-time limit, for quite generic initial configurations of the system, one never sees an equilibrium regime characterized by time translational invariance of the correlation function and the satisfaction of the fluctuation dissipation theorem [2, 3]. Recently the authors presented a detailed examination of the dynamics of the spherical Sherrington–Kirkpatrick (SK) spin-glass model [4] in [5], hereafter referred as I.

In general the initial conditions do not lead the system to equilibrium and the correlation function was shown to exhibit the aging phenomenon. This model joins a whole range of models where non-equilibrium behaviour is present, however the precise physical mechanisms responsible for aging in various models can be completely different but still give the same mathematical behaviour for various dynamical functions. Aging can arise due to the existence of large energy barriers in the system (see e.g. [6, 7]), also due to the existence of zero modes in the free-energy landscape as in the case of the models considered in [5, 8], due to a combined effect of large energy barriers and extended flat regions in the energy landscape as in mean-field models [2, 3, 9], via domain growth or coarsening mechanisms in the system [10] and even due only to entropic reasons [11]. In all these cases the correlation functions exhibit aging effects. However, the domain growth as well as the pure zero-mode mechanisms do not seem to be sufficient to capture aging in quantities such as the magnetization. The effects of small perturbations in such models are rapidly erased and hence the response decays too fast to reproduce the slow relaxations of glassy systems.

In this paper we revisit the spherical SK model but with the addition of a quenched magnetic field. For simplicity we work with a random magnetic field but this does not change the underlying physics of the problem. The theory of linear response applied to this model would suggest that, for sufficiently small fields, the behaviour of the system is not drastically altered and that one should recover the type of aging phenomena observed in I. However, the random field strength appears coupled to a *nonlinear* term in the equation

for the dynamical Lagrange multiplier imposing the spherical constraint and hence it is not apparent that linear response theory should hold. We shall concentrate on the case of zero temperature where the discussion of both the statics and dynamics is most clear. This is, however, sufficient to bring out the salient points of the problem. One finds that the random magnetic field introduces a time scale into the problem. Below this characteristic time one indeed sees aging phenomena and the effects seen in I are reproduced. Above this timescale the system does indeed *reach* an equilibrium state. With the benefit of this insight we carry out a simple analysis of the energy landscape and point out that it is the disappearance of the preponderance of zero modes in the energy landscape which is responsible for the ultimate equilibrium behaviour, adding weight to the assertions made in I.

We shall recall briefly the definition of the model. The Hamiltonian is given by

$$H = -\frac{1}{2} \sum_{ij} J_{ij} s_i s_j - \sum_i h_i s_i \quad (1)$$

subject to the spherical constraint $\sum_i s_i^2 = N$ (N being the number of spins). The matrix J is a random symmetric matrix with independently Gaussianly distributed components scaled with N to give the Wigner semi-circle law distribution for the eigenvalues μ in the thermodynamic limit, that is

$$\rho(\mu) = \frac{1}{2\pi} \sqrt{4 - \mu^2} \quad \mu \in [-2, 2]. \quad (2)$$

The terms h_i represent the random field which is chosen to be Gaussian with zero mean and variance h^2 . The dynamical equation for the evolution of the spins is then given by (at zero temperature) [5, 12]

$$\frac{\partial s_i}{\partial t} = J_{ij} s_j - z(t) s_i + h_i. \quad (3)$$

We shall assume uniform initial conditions for the spins [5] and proceed by diagonalizing the equations of motion and computing the dynamical Lagrange multiplier z self-consistently. Defining

$$\Omega(t) \equiv \exp \left(\int_0^t z(t') dt' \right) \quad (4)$$

we find it obeys the following equation

$$\Omega^2(t) = f(t) + h^2 \int_0^t dt' \int_0^{t'} dt'' f \left(t - \frac{t' + t''}{2} \right) \Omega(t') \Omega(t'') \quad (5)$$

where

$$f(t) \equiv \int \rho(\mu) d\mu e^{2\mu t}. \quad (6)$$

In contrast with the zero field case the equation satisfied by the dynamical Lagrange multiplier is now nonlinear (where as before it was a linear Volterra equation [5]). The equation (5) is evidently very difficult to solve, however it is clearly causal and accessible to numerical solution. We proceed by making the following ansatz on the asymptotic form for Ω :

$$\Omega(t) \sim ce^{\lambda t} \quad (7)$$

where c is some positive constant. Furthermore if we assume *a priori* that $\lambda > 2$ then because $f(t) \sim e^{4t}(2t)^{-3/2}/\sqrt{4\pi}$ for sufficiently large times we may make the asymptotic approximation

$$\Omega^2(t) = h^2 \int_0^t dt' \int_0^t dt'' f\left(t - \frac{t' + t''}{2}\right) \Omega(t')\Omega(t''). \quad (8)$$

Assuming the Ω is well behaved and bounded for small times we may also assume that the exponential behaviour dominates the double integral to obtain the following equation determining λ :

$$1 = h^2 \int d\mu \frac{\rho(\mu)}{(\lambda - \mu)^2} = \frac{h^2}{2} \left(-1 + \frac{\lambda}{\sqrt{\lambda^2 - 4}} \right). \quad (9)$$

The equation (9) yields the solution

$$\lambda = \frac{2 + h^2}{\sqrt{1 + h^2}} \quad (10)$$

hence we see *a posteriori* that indeed $\lambda > 2$ for $h > 0$. Of course the fact that we have found an asymptotic solution for equation (5) does not mean that it is the solution that matches with the given initial conditions (the problem of matching in these kinds of systems is, in general, outstanding, e.g. see [2, 9]). However, we have checked numerically that this is indeed the case. The results of a numerical integration of equation (5) for $h = 0.5$ and $h = 0.4$ are shown in figures 1 and 2 respectively. The function plotted is actually $\omega(t) = e^{-2t}\Omega(t)$. The figures clearly show the onset of an exponential behaviour of $\omega(t)$ around the characteristic time scale $\tau(h)$, with the analytically predicted rate (plotted as the broken curve); also shown is the zero field result (dotted curve).

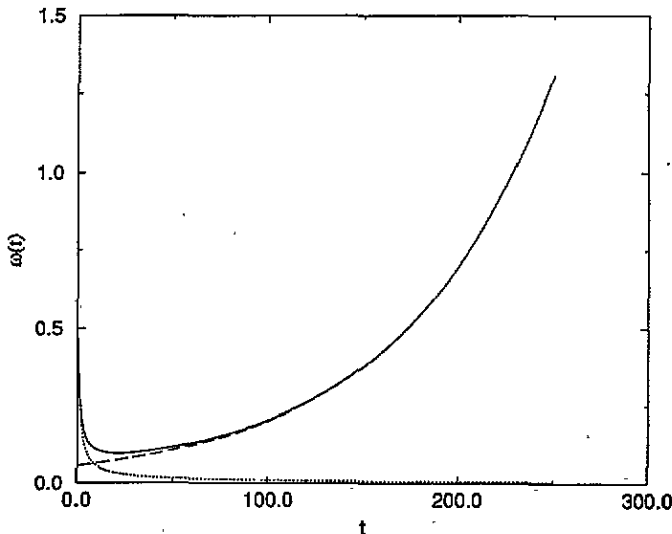


Figure 1. Plot of $\omega(t)$ against time for $h = 0.5$.

Hence we see via the above analysis that the addition of the random magnetic field introduces a time scale τ into the problem such that for times sufficiently large compared to τ the contribution of the first term of the right-hand side of equation (5) becomes negligible compared to the second term. (It is, in fact, the first term that contains all the information

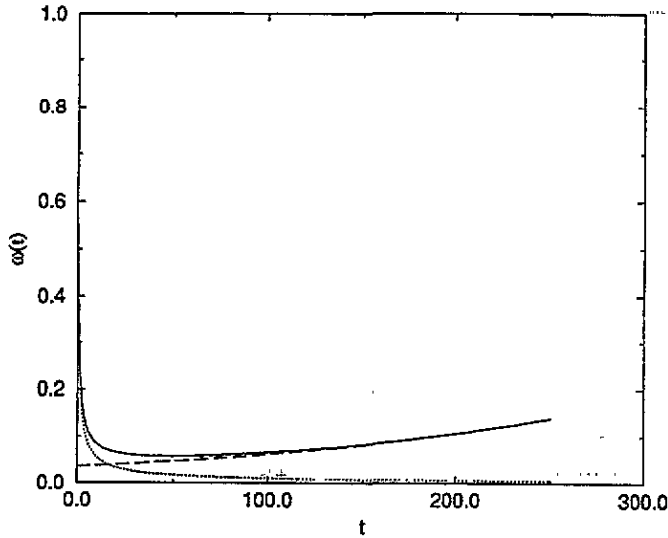


Figure 2. Plot of $\omega(t)$ against time for $h = 0.4$.

about the initial condition; the fact that it becomes negligible demonstrates that the initial condition is completely forgotten at large enough times.) The analysis in addition shows that

$$\tau \sim 1/(\lambda - 2) = \left(\frac{2 + h^2}{\sqrt{1 + h^2}} - 2 \right)^{-1} \quad (11)$$

which clearly diverges when $h \rightarrow 0$ as h^{-2} . However, for sufficiently small fields and/or times it is the first term of the right-hand side of equation (5) which dominates, and this leads to the aging phenomena exhibited in [5] within these field/time regimes.

The explicit demonstration of equilibrium behaviour for sufficiently large times is now rather trivial. The correlation function is given by

$$C(t, t') = \frac{1}{\Omega(t)\Omega(t')} \left[f\left(\frac{t+t'}{2}\right) + h^2 \int_0^t ds \int_0^{t'} ds' f\left(\frac{t+t'-s-s'}{2}\right) \Omega(s')\Omega(s) \right]. \quad (12)$$

Inserting the asymptotic form for Ω simply yields

$$C(t, t') \sim 1 \quad (13)$$

for sufficiently large times. Hence the system ultimately reaches its equilibrium state, which is simply the point of minimal energy in the zero temperature case.

The energy density $\mathcal{E}(t)$ can be shown to obey

$$\mathcal{E}(t) = -\frac{1}{2} \frac{d}{dt} \log(\Omega) - \frac{1}{2} \alpha(t) \quad (14)$$

where α is the average spin/field correlation

$$\alpha(t) = \frac{1}{N} \sum h_i s_i(t) = \frac{h^2}{\Omega(t)} \int_0^t dt' f\left(\frac{t-t'}{2}\right) \Omega(t'). \quad (15)$$

One finds that

$$\mathcal{E}(t) \rightarrow -\sqrt{1 + h^2} \quad (16)$$

and

$$\alpha(t) \rightarrow \frac{h^2}{\sqrt{1+h^2}}. \quad (17)$$

Now we carry out a simple analysis of the geometry of the energy landscape for the model in the presence of a random magnetic field. The energy density corresponding to a spin configuration s is given by

$$N\mathcal{E} = -\frac{1}{2}s \cdot Js - h \cdot s + \lambda \frac{1}{2}(s \cdot s - N) \quad (18)$$

where λ is the (static) Lagrange multiplier imposing the spherical constraint. The variational equations yield the stationary solutions

$$s = (\lambda - J)^{-1}h \quad (19)$$

where λ satisfy the equation

$$\frac{1}{N}h(\lambda - J)^{-2}h = h^2 \int \frac{\rho(\mu)}{(\lambda - \mu)^2} d\mu. \quad (20)$$

For $\lambda \in [-2, 2]$ the integral above diverges, and hence for finite h we must look for solutions outside this interval. There are only two and are given by

$$\lambda_{\pm} = \pm \frac{2+h^2}{\sqrt{1+h^2}}. \quad (21)$$

The corresponding values of the energy density can be shown to be

$$\mathcal{E}_{\pm} = -\frac{1}{2}h^2 \int \frac{\rho(\mu)}{(\lambda_{\pm} - \mu)} d\mu - \frac{1}{2}\lambda_{\pm}. \quad (22)$$

The minimum of the energy is given by λ_+ , and λ_- gives the maximum energy (which must also exist according to Morse theory). The minimum energy and the static value of α are in agreement with the dynamically calculated limits in equations (16) and (17).

If one analyses the zero magnetic field case one finds that there are N stationary points for the energy, each one corresponding to an eigenvector of the interaction matrix. Hence the effect of the field statically has been to reduce the number of stationary points from a macroscopic number down to just two. (This is in sharp contrast to the reduction in the number of metastable states in the SK model by a magnetic field, where the number always remains macroscopic for finite fields [13].) Therefore from almost every starting configuration (except the maximum which is, however, measure zero), the system has a drift towards the equilibrium state. It was postulated in [5] that the large number of flat regions in the zero field landscape was responsible for the aging phenomena observed. Here we see clearly that once these are eradicated an equilibrium state is achieved.

We have analysed the influence of a magnetic field on the dynamics of the spherical SK spin-glass. At variance with the prediction of linear response, the aging behaviour of the system is completely eradicated after a characteristic time which decreases with increasing field strength. The failure of linear response is presumably due to the inherent nonlinearity introduced in the equation of motion for the dynamical Lagrange multiplier. However, for sufficiently short times, linear response is presumably valid and one does indeed see that below this time scale the aging phenomenon is present. In fact this model explicitly demonstrates the phenomenon of interrupted aging discussed in [14].

Also we have related the ultimate arrival at equilibrium of the system to the geometry of the energy landscape of the model, showing that the field reduces the number of stationary (flat) points in the energy landscape from a macroscopic number to just two. This is in

agreement with the notion that in the spherical SK spin-glass in the absence of a random magnetic field, it is the zero modes in the system which give rise to the aging phenomenon rather than tunnelling through large energy barriers.

The behaviour of the spherical SK model is reminiscent of what is predicted by the scaling approach to spin-glasses [15]. Within the droplet picture, the low-temperature phase is characterized by only two pure states as for the spherical SK model. The other hallmark of the droplet picture is the absence [16] of a genuine h - T transition line (AT line [17]) and hence the destruction of the thermodynamic spin-glass phase by any applied field. However, a pseudo AT line defined as the locus of points at which the system gets frozen on experimental times scales moving down with increasing time scales is claimed to exist in this approach [16]. In numerical studies [18] of the two-dimensional Ising spin-glass, despite the absence of a static phase transition and AT line, exactly the same phenomenon of a dynamical AT line has been observed. For the spherical SK model, statically there is no AT line but dynamically we find that the non-equilibrium aging behaviour exists below a transition line that moves towards zero field when the times explored increase.

Experimentally there is no clear evidence for the appearance of an equilibrium regime for spin-glasses under the influence of a magnetic field [19]. However, we expect that the behaviour observed in this letter may be present in other disordered systems such as the problem of a driven particle in a random potential [9, 20].

We would like to thank J-P Bouchaud, J Kurchan, P Le Doussal, M Mézard and E Vincent for useful discussions. DSD acknowledges support from a UK EPSRC Research Fellowship and also from EU grant CHRX-CT93-0411. LFC acknowledges support from the EU HCM grant ERB4001GT933731.

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